

## Learning Objectives for MA 266:

1. Students will know these definitions and concepts, and be able to apply and interpret them in context of relevant application problems:
  - a. Ordinary differential equation, mathematical model, initial condition, initial value problem, order, linear, nonlinear, general solution, particular solution.
  - b. Separable equations, including: implicit solution, singular solution, half-life.
  - c. Nonlinear equations, including: Homogeneous equations, Bernoulli equation, exact differential equations, differential form, reducible second-order equations.
  - d. General population equation, logistic equation, limiting population, carrying capacity, threshold population.
  - e. Autonomous equations, equilibrium solutions, critical points, stable, unstable, harvesting, bifurcation point, bifurcation diagram.
  - f. Numerical approximation: including step size, Euler's method, improved Euler's method
  - g. Linear equations, including: homogeneous, nonhomogeneous, principle of superposition, linearly independent, linearly dependent, Wronskian, repeated roots, complementary solution.
  - h. Homogeneous equations with constant coefficients, including: characteristic equation, Euler's formula, complex-valued functions, real, imaginary, derivative of a complex function, polar form of complex numbers.
  - i. Mechanical vibrations, including: mass-on-a-spring, equilibrium position, spring constant, damping constant, damped, undamped, external force, free motion, forced motion, simple harmonic motion, period, frequency, time lag, underdamped, overdamped, critically damped, circular frequency, pseudoperiod, time-varying amplitude, natural frequency, beats, resonance, transient solution.
  - j. Systems of differential equations, including: solution curve, trajectory, linear systems, homogeneous, nonhomogeneous.
  - k. Matrix algebra, including matrix multiplication, inverses, determinants, eigenvalues and eigenvectors, multiplicity, defective eigenvalue, and generalized eigenvector.
  - l. Linear systems of equations, including: homogeneous, nonhomogeneous, the principle of superposition, linear independence, Wronskian, Jacobian matrix, fundamental matrix, exponential matrix.
  - m. Phase portraits, including: phase plane, proper node, improper node, saddle point, spiral point, center, source, sink.
  - n. Laplace and inverse Laplace transform, including: piecewise continuous function, unit step function, convolution, impulse, delta function, transfer function, weight function, and Duhamel's principle.
2. Students will be able to describe, sketch, analyze, set up, and implement on paper the following procedures and calculations:
  - a. Solve an initial value problem.
  - b. Solve first and second order ODE's using direct integration.
  - c. Construct a slope field for a first order ODE and interpret the solution.
  - d. Solve a separable equation using the method of separation of variables.
  - e. Solve a linear first order equations using the integrating factor.
  - f. Solve a differential equation using substitution methods.

- g. Solve an exact differential equation.
  - h. Evaluate the dynamics of a given population model.
  - i. Determine the stability of the critical points of a given equation.
  - j. Find an approximate solution to an ODE using Euler's method and/or improved Euler's method. Calculate by hand and by computer code.
  - k. Solve a second order linear equations with constant coefficients.
  - l. Solve a homogeneous linear equation using the method of reduction of order.
  - m. Solve a nonhomogeneous linear equation using the method of undetermined coefficients and/or variation of parameters.
  - n. Solve a system of equations using the method of elimination.
  - o. Solve a system of linear differential equations using the eigenvalue method.
  - p. Identify and draw a direction field and phase portrait at a critical point.
  - q. Determine the stability of a system of ODEs at a critical point.
  - r. Compute the matrix exponential solution of a linear system.
  - s. Solve a nonhomogeneous linear system using the method of undetermined coefficients and/or variation of parameters
  - t. Take the Laplace transform of a piecewise continuous function.
  - u. Solve initial value problems using the Laplace transform.
3. In particular, students should be able to model the following applications using the appropriate differential equation and effectively communicate the outcome:
- a. Growth and decay models, including: population growth, compound interest, radioactive decay, drug elimination, cooling and heating.
  - b. Mixture problems.
  - c. Population models.
  - d. Acceleration-Velocity models.
  - e. Mass-on-a-spring problem.
  - f. Simple pendulum.